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Disorder-driven crossover from first-order to second-order depinning

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Abstract. It is shown that an extra defect line introduced into a solid-on-solid model to describe a second-order wetting transition in two dimensions may lead to a sharp first-order depinning. If disorder is introduced along the defect line, this first-order transition may be driven to second order. If the transition in the pure system is second order, disorder may drive it to first order. All the calculations leading to the above results can be performed analytically.

The effect of disorder on a second-order phase transition can be accessed using the Harris criterion (Harris 1974). Imry and Wortis generalised Harris' argument with the following result: under appropriate conditions disorder may turn a first-order transition to second order (Imry and Wortis 1979). The validity of this conclusion has since been demonstrated experimentally (Ryan *et al* 1986). The aim of this publication is to present for the first time an analytic calculation supporting the result of Imry and Wortis.

We consider wetting phenomena in two dimensions (2D). It has been shown (Forgacs *et al* 1988b) that the introduction of an additional defect line of bonds (see below) in Abraham's model (Abraham 1980) for continuous wetting transition can lead to a sharp first-order unbinding transition. The importance of this result stems from the earlier belief that discontinuous unbinding can take place only in the presence of long-range interactions. No such interactions are present in the model studied by Forgacs *et al* (1988b), which uses the language of Ising spins.

As a separate recent development, the effect of disorder introduced on the wall or substrate (from which unbinding takes place) has been studied using mean-field considerations (Forgacs *et al* 1985), Monte Carlo calculations (Speth 1986) and real-space renormalisation (Svrakic 1985). Another approach is via the restricted solid-on-solid (RSOS) model (Forgacs *et al* 1986, 1988a). It was demonstrated that, provided the transition in the pure system is of second order, substrate randomness is not a relevant perturbation. It introduces 'only' logarithmic corrections to the pure system's free energy, but the critical exponents are unaffected. Moreover, the condition of criticality of the quenched random system is exactly the same as that of the annealed random system (Forgacs *et al* 1986, 1988b). These results should be contrasted with those for bulk randomness. If bulk randomness is uncorrelated (Kardar 1985, Lipowsky and Fisher 1986), the critical exponents are changed with respect to the pure system. If bulk randomness is fully correlated in the direction of the wall the transition becomes first order, despite the absence of long-range forces (Nieuwenhuizen 1988).

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In the present work, by combining the above results we are able to prove that substrate-like disorder indeed may turn a first-order unbinding transition to second order. For this purpose, we first reproduce the results on the first-order unbinding (Forgacs *et al* 1988a), mentioned above, in terms of the RSOS model. These results will be needed when disorder is introduced.

Consider the 2D RSOS model

$$H = J \sum_{i=1}^{N} |h_i - h_{i+1}| - u \sum_{i=1}^{N} \delta_{h_i,0} - v \sum_{i=1}^{N} \delta_{h_i,L}. \tag{1}$$

The difference of nearest-neighbour height variables h_i (h_i assume integer values between 0 and ∞) may be only $0, \pm 1$. J, u and v are positive. If $v = 0$ (model A), (1) describes a continuous wetting transition (Chui and Weeks 1981, van Leeuwen and Hilhorst 1981). The parameter u models an attractive wall potential at $h_i = 0$ and for zero temperature, $T = 0$, the interface is localised at this substrate. (In the RSOS model $h_i = n$ means that in the ith column n down-spins are followed by up-spins and the interface develops between the regions of up- and down-spins. The model gives an accurate description of the low-temperature phase of the Ising model.) This interface depins smoothly at a finite temperature T_W, where $\langle h_i \rangle = \infty$ ($\langle\ \rangle$ denotes the thermodynamic average).

For $u = 0$, $v \neq 0$ at $T = 0$ the interface is localised at a defect line at a distance L from the wall. The mechanism of depinning is the same as in model A. If $L \to \infty$ (after $N \to \infty$) for finite v, depinning from the line takes place only at infinite temperature. We call this ($u = 0$, $v \neq 0$, $L \to \infty$) model B. Model B is then equivalent to a symmetric model, in which the h_i take on integer values between $-\infty$ and $+\infty$; the defect line with v is placed at $h_i = 0$. Let us now consider the general case $u \neq 0$, $v \neq 0$. If $u > v$, at $T = 0$ the interface is still pinned to the substrate. For finite L, at a finite temperature (although shifted relative to T_W) there is still a second-order wetting transition. In the limit $L \to \infty$ ($L < N$), however, at some $T_F < T_W$ the interface unbinds from the substrate via a sharp first-order transition and becomes pinned to the defect line in the bulk ($L = \infty$). In the RSOS model this first-order depinning at T_F is followed by a continuous wetting transition (in which the interface depins from the bulk defect line) at $T = \infty$. In the Ising model this continuous transition occurs at the Onsager critical point (Forgacs *et al* 1988b, c).

In order to calculate the partition function corresponding to (1) in the limit $L \to \infty$ we use the transfer matrix method. The spectrum of the transfer matrix consists of a continuous part and a pair of discrete eigenvalues. The latter, denoted by λ_A, λ_B, are given by the solutions of

$$(y + yt\, e^{-\mu} - \lambda)(w + 2wt\, e^{-\mu} - \lambda) = 0. \tag{2}$$

Here $t = \exp(-J/T)$, $y = \exp(u/T)$, $w = \exp(v/T)$ and μ is a function of λ itself given by

$$\lambda = 1 + 2t \cosh \mu. \tag{3}$$

The solutions $\lambda_A(y)$ and $\lambda_B(w)$ of (2) separately determine the partition functions of models A and B (with $L \to \infty$) respectively. Note the difference between λ_A and λ_B is due to the factor of two in (2). The physical origin of this factor is related to the possibility of the h_i to fluctuate both in the $h_i < 0$ and $h_i > 0$ half-spaces in the symmetric model (see above). Loosely speaking, the h_i in the symmetric model fluctuate in a space 'twice bigger' than in model A.

λ_A and λ_B, whenever they exist, are larger than any eigenvalue of the continuous spectrum of the transfer matrix. The free energy of the system, given by the largest eigenvalue of the transfer matrix is determined then by (2). The first-order transition temperature is given by $\lambda_A = \lambda_B$, i.e.

$$y(1+t_F\,e^{-\mu}) = w(1+2t_F\,e^{-\mu}) \tag{4}$$

where $t_F = \exp(-J/T_F)$. If $y > w$ $(u > v)$, the free energy (per site) is

$$-f/T = \begin{cases} \ln \lambda_A = \ln[y(1+t\,e^{-\mu})] & T \leqslant T_F \\ \ln \lambda_B = \ln[w(1+2t\,e^{-\mu})] & T \geqslant T_F. \end{cases} \tag{5}$$

These results are summarised in figure 1. The free energy of our model ($u \neq 0$, $v \neq 0$) in the $L \to \infty$ limit is given by the full curve. It has a cusp at t_F. The full curve for $t < t_F$ and broken curve for $t > t_F$ represent the free energy of model A. The other analytic (broken) curve for $t < t_F$ and full curve for $t > t_F$ represent the free energy of model B. When $w = 1$ $(v = 0)$ we have

$$t_F(w=1) = t_W = \exp\left(-\frac{J}{T_W}\right) = \frac{y-1}{2-y}. \tag{6}$$

Our discussion until now has been aimed at reproducing the results obtained previously for the Ising model (Forgacs *et al* 1988b) within the framework of the RSOS model. We now consider what is the effect of disorder on the first-order depinning transition. Only substrate-like disorder will be considered here. It will be shown that if v is chosen to be a random independent interaction at each site, the first-order transition, discussed above, may be driven to second order in agreement with the conclusions of Imry and Wortis (1979). Let us consider a model with the third term in (1) replaced by $\Sigma_i v_i \delta_{h_i,L}$, where v_i (for each i) has a probability distribution

$$P(v_i) = p\delta(v_i - v_A) + (1-p)\delta(v_i - v_R). \tag{7}$$

Here $v_A > 0$ and $v_R < 0$ correspond to attractive and repulsive defect potentials, respectively. When $u = 0$ (model B, since $L \to \infty$) in the pure system ($p = 1$ in (7)) continuous

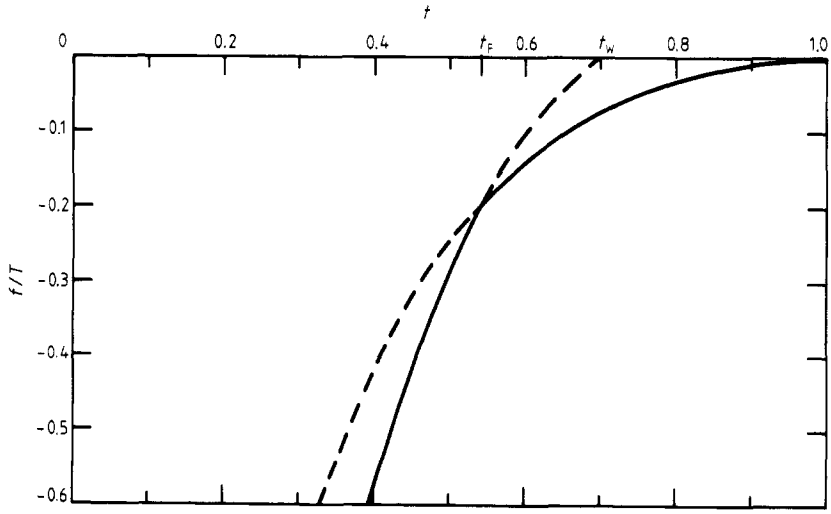

Figure 1. Free energies of models A and B and of the model defined by (1). For more details see text. The curves are drawn for $v = 0.9$, $u = 1.2$, $J = 1$.

depinning occurs only at $T=\infty$, $w=1$. As was shown by Forgacs *et al* (1986, 1988a) in the quenched random case ($p<1$) with $u=0$ this condition is replaced by

$$\bar{w}=\int \mathrm{d}v\, P(v)\exp(v/T_{\mathrm{D}})=1. \tag{8}$$

Here, the depinning temperature T_{D} is finite only if $\bar{v}<0$. Equation (8) is also the condition of critical depinning in the corresponding annealed system (Forgacs *et al* 1986, 1988a). The free energy of the quenched disordered version of model B is

$$\frac{f_{\mathrm{B}}-f_{w}}{T}=\frac{\mu^2 t}{1+2t}\left(S+\frac{2\pi}{\ln(1/\mu)}\right)+\mathrm{O}(\mu^3). \tag{9}$$

Here f_{w} is the free energy of the unbound system and is given by $-f_w/T=\ln(1+2t)$. S in (9) is a non-universal constant which depends on the form of $P(v)$. The above equation is valid for small μ, in the vicinity of T_{D}, for $T\leqslant T_{\mathrm{D}}$. The actual calculation of f_{B} can be summarised as follows. (For details see Forgacs *et al* (1986, 1988a) where the random versions of model A and of the symmetric model are solved explicitly.) To treat randomness one uses the replica trick. This leads to an effective many-body Hamiltonian with n-body interactions ($n=2,3,\ldots$). The coupling constant of the n-body interaction is proportional to the nth cumulant of (7). The effect of the many-body interactions is calculated in perturbation theory. It turns out that interactions with $n>2$ are all irrelevant and do not contribute to the singular part of the free energy. All terms of the perturbation series with $n=2$, on the other hand, can be summed up exactly (in the vicinity of the continuous wetting transition), leading to the logarithmic term in (9). (The validity of the replica trick has been checked numerically.)

Starting with a pure system $p=1$, $u\neq 0$, $v\neq 0$ (in the $L\to\infty$ limit) and then decreasing p in (7) we expect the first-order depinning transition of the pure system to disappear at a given value of p. This will happen when T_{D} coincides with T_{W}, since in this case the free energies of the pure model A and the random model B do not intersect anymore. These observations can be expressed in an analytic form for $(T_{\mathrm{W}}-T_{\mathrm{D}})/T_{\mathrm{W}}\ll 1$ when (9) is valid. For larger deviation from T_{W} we have calculated the free energy of the full system ($u\neq 0$) numerically. The phase diagram is depicted in figure 2 for $J=1$, $u=0.3$, $v_{\mathrm{A}}=1$ and $v_{\mathrm{R}}=-4$.

The pure system with attractive u and all $v_i=v_{\mathrm{A}}$ corresponds to $p=1$. The interface is pinned to the defect line already at $T=0$, and depins from it only at $T=\infty$. This is the consequence of our choice $v_{\mathrm{A}}>u$. The same behaviour occurs for $p_{\mathrm{d}}<p<1$, where p_{d} is defined by

$$\bar{v}=p_{\mathrm{d}}v_{\mathrm{A}}+(1-p_{\mathrm{d}})v_{\mathrm{R}}=0. \tag{10}$$

Below this concentration the defect line is repulsive on the average and there is a continuous wetting transition at finite T_{D}. This is the only transition in the interval $p_1<p<p_{\mathrm{d}}$. Here p_1 is the value of p at which the zero-point energy of the state bound to the wall ($-u$) equals the zero-point energy of the state pinned near the defect line. Actually, at $T=0$ the latter energy is determined by an interface, which is partially at $h_i=L$ (in order to benefit from the attractive regions) and partially at one of the degenerate positions $h_i=L\pm 1$ (in order to avoid repulsive regions of the defect line). Thus, the problem at $T=0$ is equivalent to a random field Ising chain at $T=0$ (Forgacs *et al* 1988a), and the zero-point energy follows from the calculation of Derrida *et al* (1978). For p in the region $p_{\mathrm{c}}<p<p_1$, there is a sharp first-order depinning transition

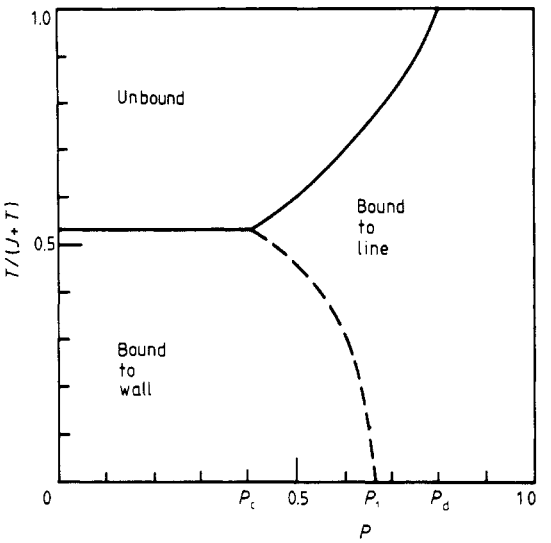

Figure 2. Phase diagram of the disordered model. For more details see text.

at finite T_F, where the interface jumps from the wall to the defect line. This transition is followed by a continuous wetting transition, where the interface unbinds from the defect line. p_c follows from the equality $T_F = T_W$, where T_W is the wetting temperature of the system without defect line at L (model A), and is given by (6). With the values of J and u, used in the numerical calculation, (6) leads to $T_W/(1+T_W) = 0.53$ in figure 2. The equality $T_F = T_W$ (T_F determined by (4) with w replaced by $\bar{w}$) then gives $p_c = 0.4$. Finally, for $0 < p < p_c$, the interface does not bind to the defect line at all, essentially because it is too repulsive.

Throughout this work we have carefully avoided calling the first-order transition a wetting transition. In fact in Cahn's terminology it is not a wetting transition (Cahn 1977); Antonov's rule for the contact angle does not hold (Forgacs *et al* 1988c).

In conclusion, we have shown that the effect of a disordered defect line in the bulk of a 2D RSOS model is to completely change the character of the unbinding transition. Such a line, depending on the strength of the disorder, may lead to a first-order depinning transition or may turn the first-order transition already existing in the pure system to second order in agreement with the result of Imry and Wortis (1979).

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